Jobless Objects: Mathematical Posits in Crisis Yvonne Raley

Abstract:

This paper focuses on an argument against the existence of mathematical objects called the "Makes No Difference Argument" (MND). Roughly, MND claims that whether or not mathematical objects exist makes no difference, and that therefore, we have no reason to believe in them. The paper analyzes four different versions of MND for their merits. It concludes that the defender of the existence of mathematical objects (the mathematical Platonist) *does* have some retorts to the first three versions of MND, but that *no* adequate reply is possible to the fourth, and most crucial, version of MND. That version argues that mathematical objects make no difference to our epistemic processes: they play no role in the process of obtaining mathematical knowledge.

I. Introduction

As the debate over the existence of mathematical objects continues, a "new" type of argument has surfaced as a challenge to mathematical Platonism. Alan Baker (2003) calls it the "Makes No Difference" argument (MND). Roughly, the "Makes No Difference" argument says that whether or not mathematical objects – objects that are said to be neither spatial nor temporal, and that are causally inert - exist makes no difference, and that therefore, we have no reason to believe in them.

How is this lack of difference-making cashed out? Here, the literature offers various options. Horgan (1987: 281, 282) has it that the "world's spatio-temporal causal nexus" would be unaltered if sets did not exist. For Ellis (1990: 113), "the world we can know about" would be the same if there were no abstract objects. Azzouni (1994: 56) believes that if "mathematical objects ceased to exist", … "[m]athematical work" would "go on as usual." For Balaguer (1999: 113), "if there were no such things as abstract objects, science would be practiced exactly as it is right now". And lastly, Baker (2003: 247) describes MND as saying that "[i]f there were no mathematical objects, then … this would make no difference in the concrete, physical world."[1]

Given this family of similar, but far from identical, formulations of MND, tracing the argument to a point of origination is difficult. But there's another reason for this. The Makes-No-

[1] Not everyone on this list endorses the Makes-No-Difference argument, however.

Difference argument is not actually *one* argument, and the quotations provided in the previous paragraph give us a first indication of this. For what it is that mathematical objects are supposed to make a difference *to*, depends very much on the purported "job" mathematical objects are taken to have in the first place.

As I see the matter, there are four distinct arguments that hide beneath the umbrella of MND; and they need to be disentangled. That is what I propose to do here. In separating out these four versions of MND, I am especially interested in the replies available to the defender of the existence of mathematical objects – the mathematical Platonist. The dialectic that will emerge is this: Initially, the Platonist's position will actually look quite promising, as he has at least some responses to the first three versions of MND (sections II, III, and IV). However, as we investigate further how the claim that mathematical objects make no difference might be understood, we will see that the first three versions of MND don't really get at the heart of the debate. It is only the fourth and last version of MND that succeeds in locating the focal point of the issue: whether or not mathematical objects have an *epistemic* role (section V). In my view, when it comes to the question of whether or not mathematical objects play a role in the process of obtaining mathematical knowledge, the Platonist loses his initial advantage. Such objects *don't* play a role. Consequently, whether or not mathematical objects exist really makes no difference. Thus the proponent of MND wins: without an epistemic role for mathematical objects, we have no reason to believe in their existence.

II. MND and Causal Inertness

The first version of MND is based on the assumption that mathematical objects are causally inert. This assumption is well-entrenched in the philosophy of mathematics literature and it is sometimes called "the standard platonistic view." For example, Cheyne and Pigdon (1996: 639) "take the standard platonistic position to include the claim that platonic objects lack spatio-temporal location and causal powers."

The fact that mathematical objects are understood to be causally inert lends itself to a quick and dirty version of MND: If the objects of mathematics are causally inert, then they cannot affect any objects or processes in the concrete-physical world. Therefore, we have no reason to believe in their existence. So, in this version of MND, to make a difference, a mathematical object has to be causally effacatious.

Strands of this argument are discussed in Horgan (1987: 281, 282): "Since sets are not supposed to be part of the world's spacio-temporal causal nexus, that nexus would be exactly as it is whether sets existed or not…," and in Balaguer (1998: 133):

> Empirical science *knows*, so to speak, that mathematical objects are causally inert. That is, it does not assign any causal role to any mathematical entity. Thus, it seems that empirical science *predicts* that the behavior of the physical world is not dependent in any way upon the existence of mathematical objects.[2]

How can a Platonist respond to this version of MND? In my view, the best response (and probably the most common) is to simply concede the point. If mathematical objects are causally inert, then *of course* they can't affect any objects or processes in the physical world. (That's why I called this version of MND the "quick and dirty version"). But, the Platonist can go on to say, that this is not the sort of role he had in mind for his objects in the first place. After all, it is a role he has "defined away" by his own assumptions.

[2] I don't know how Balaguer thinks he's established what empirical science knows, or even what it "*knows*, so to speak" about the causal inertness of mathematical objects (who knows this, how do they know it?). But, as I said above, the view is certainly presumed in the philosophical literature, which is all we need for our purposes.

The Platonist therefore has to take a different approach to delineating a role for mathematical objects. To see what he might have to offer, we have to go on a short tangent and explore a proposed counter-example to the view that mathematical objects are causally inert. A detailed evaluation of this counter-example will reveal a possible role for mathematical objects that avoids the issue of their nature. It will also lead us to a second version of MND.

The counter-example I have in mind is due to Cheyne and Pigdon (1996). Cheyne and Pigdon, in their paper, focus on the question of whether the Quine-Putnam indispensability thesis establishes standard Platonism about mathematics. The indispensability thesis is cashed out as the claim that "mathematical objects are just as indispensable to science as theoretical entities like electrons. Electron theory quantifies over numbers, just as it quantifies over electrons" (1996: 640).

Cheyne and Pigdon then claim that even if we grant that mathematical objects are indispensable to science (which they *do* grant), then standard Platonism is in trouble. Here's why:

> If we are genuinely unable to leave those objects out of our best theory of what the world is like (at least, that part of the world with which we causally interact), then they must be responsible in some way for that world's being the way it is. In other words, their indispensability is explained by the fact that they are causally affecting the world, however indirectly. (1996: 641)

As is well known, and as Cheyne and Pigdon correctly observe, the indispensability thesis is a claim about what sorts of objects our best scientific theories need to quantify over. It then says that whatever objects we do quantify over are objects that we have to take as real. In short, what it means for an object to be indispensable to science is for that object to essentially fall within the range of the (objectual) existential quantifier once our best theories are written down.

But how does indispensability, which just involves quantification, bring causal powers to the objects so quantified over? Cheyne and Pigdon offer support for this view with the following example:

> The fact that there are three cigarette butts in the ashtray causes Sherlock to deduce that Moriarty is the murderer, and that if there had been more or fewer butts he would have deduced otherwise. (1996: 642)

Cheyne and Pigdon then say that the fact that there are three cigarette butts in the ashtray is causal, and that "the number three is an indispensable constituent of this fact" (1996: 642).[3] Furthermore, they say that the number three makes a causal difference because had there been a different number of cigarettes in the ashtray, different effects could be expected (1996: 642).

Cheyne and Pigdon want to use this example to challenge to the standard Platonist like this: "Our challenge to Platonists is for them to provide an explanation for the indispensability of objects whose presence (they claim) makes no causal difference" (1996: 642). But of course, that challenge has long been met, because we already *have* an explanation for the indispensability of mathematical objects: mathematical objects are indispensable because we cannot rewrite the language of science in such a way that avoids quantification over them! So the Platonist need not explain anything. The indispensability thesis is a claim about what sorts of entities have to be quantified over in (the best version of) scientific discourse. It is *not* a claim about what sorts of entities are causally efficacious. The indispensability thesis, per se, does not require the indispensable objects to have any particular properties. The properties they have are dictated by the scientific discourse. And, when it comes (say) to numbers, there isn't anything in scientific discourse that requires them to be causal.

[3] In a footnote, Cheyne and Pigdon are quick to point out that "[the number three] isn't, of course" (indispensable). They believe that more likely cases of indispensability will be found in General Relativity or quantum mechanics.

There's another point to be raised here. As footnote 3 states, in the example Cheyne and Pigdon offer, the number three is not actually indispensable – the number three need not be quantified over. Now compare the example Cheyne and Pigdon offer with an example that involves a universal such as "redness." In many sentences, such universals are also not quantified over, e.g.: "The fact that Lucinda wore a red dress causes Sherlock to deduce that Moriarty is the murderer" (and if Lucinda had worn a different color dress, he would have deduced otherwise). If, by analogy to Cheyne's and Pigdon's example, we further suppose that this fact is causal, and that the redness of Lucinda's dress is a constituent of this fact, wouldn't we then have to say that the property of redness exists even though it is *not* quantified over?[4]

If that is what we say, then the indispensability of an object (or a universal, what have you) is not necessary for a commitment to the existence of that thing. That is, quantification over a thing isn't necessary to be committed to it. But if (instead) we say that since redness is not quantified over, we are *not* committed to its existence, then (in any case) its being causally efficacious isn't crucial to what we are (or aren't) ontologically committed to. The upshot: the indispensability of mathematical objects and the question of whether they have causal powers (or not) are entirely independent from one another.

III. MND and Indispensability

Having thus separated questions about indispensability from questions about the causal inertness of mathematical objects, we can now see our way to another version of MND. This version relies on the indispensability thesis directly, and avoids any questions about the nature of mathematical objects. This version of MND surfaces in Baker (2003: 254): "If mathematics is dispensable for science, then (No-Difference) is true..." Baker understands the indispensability thesis in the same

[4] This is *precisely* what Quine was trying to avoid when he used his criterion to argue *against* McX's claim that there *are* universals. (Quine, 1948: 10.)

way Cheyne and Pigdon do, namely as the thesis that science "cannot be formulated without quantifying over mathematical objects" (2003:258).[5] So our indispensability version of MND might therefore read: if scientific discourse can be formulated without quantifying over mathematical objects, then these objects do not play a role in science. Their existence does not make a difference.

But *can* scientific discourse be formulated without quantifying over mathematical objects? The answer to that seems to be "no." And Cheyne and Pigdon (among many others) have explained why: the best effort (so far) to nominalize mathematics began and ended with Newtonian physics. Famously, Field (1980) tried to show that we need not quantify over numbers in Newtonian physics (which is, however useful, not even true), and even if we grant that he was successful, this leaves Quantum Mechanics and General Relativity untouched. Therefore, our best scientific theories are stuck with quantification over numbers, and other mathematical abstracta.

On this count, therefore, the Platonist seems to win, for the antecedent to the indispensability version of MND cannot be established. However, there's an important objection to using the indispensability thesis to carve out a role for mathematical objects: what is indispensable to science are *quantifications*, which involve mathematical *terms*. This means that it is those *terms* appearing within the scope of the (objectual) existental quantifier in sentences that are indispensable. Hence, talk of indispensability, first and foremost, is talk about the indispensable *role* that mathematical *terms* play in scientific discourse. So, strictly speaking, it is not purported objects that are indispensable, but a kind of *language*. This is not to say that a connection between terms and objects cannot be argued for, but it requires that the Platonist avail himself of an additional tool. The way the objects come in (and the above quotations indicate this) is via Quine's criterion for

[5] Baker's example, in the passage in question, comes from general relativity.

ontological commitment. As Quine (1948: 13) has put it: "To be assumed an entity is, purely and simply, to be reckoned as a value of a variable."[6]

This is an important point, and it deserves further attention. But first, we will look at two other ways in which MND could be understood. As it turns out, once we are done examining the different strands of argument that hide beneath the umbrella of MND, Quine's criterion will surface twice more as a tool which the Platonist must avail himself in order to etch out a role for mathematical objects. We will therefore leave the work of Quine's criterion in this debate for last.

IV. MND and Mathematical Truth

Another role mathematical objects have often been assigned is that of grounding the truth of mathematical claims. In denying this role, MND might be formulated like so: whether or not mathematical objects exist makes no difference to the truth of mathematical claims. In order to evaluate the strength of this version of MND, let us look more carefully at what its proponent is up against.

While the tradition that mathematical objects ground the truth of mathematical claims can be traced all the way back to Plato, consider a more recent formulation of the idea, which is due to Benacerraf. According to Benacerraf (1973: 405), just as the sentence "There are at least three large cities older than New York" is true if and only if there really are three large cities that are older than New York, the sentence "There are at least three perfect numbers greater than 17" is true if and only if there really are at least three perfect numbers greater than 17. Thus, the truth conditions of the mathematical sentence seem to necessitate the existence of mathematical objects that bear certain

[6] A bit further down in the same paragraph, Quine connects ontological commitment to the quantifiers explicitly: "The variables of quantification 'something', 'nothing', 'everything' range over our whole ontology, whatever it may be; and we are convicted of a particular ontological presupposition if, and only if, the alleged presuppositum has to be reckoned among the entities over which our variables range in order to render one of our affirmations true."

relations to one another. On this view, in other words, the role mathematical objects play is to provide the conditions under which mathematical sentences are true.

Note that Benacerraf's own account, which I've here summarized briefly, actually relies on Quine's criterion for ontological commitment. Here's how Benacerraf spells out the truth conditions for the first sentence:

> [it] will be true if and only if the thing named by the expression replacing 'a' ('New York') bears the relation designated by the expression replacing 'R' ('1 is older than 2') to at least three elements (of the domain of discourse of the *quantifiers*) which satisfy the predicates replacing 'F' and 'G' ('large' and 'city' respectively). (1973: 405, emphasis mine.)

Benacerraf then suggests that the truth conditions of the mathematical sentence can be explicated analogously, since the two sentences have the same grammatical structure.[7] Therefore, the way in which the truth of mathematical sentences necessitates the existence of mathematical objects is via Quine's criterion. Mathematical objects, on this view, exist because the truth conditions for mathematical statements require the domains of the quantifiers in those statements to range over mathematical objects.

As just noted, this way of delineating a role for mathematical objects comes up against the same issue we encountered in the previous section: it is Quine's criterion for ontological commitment that connects the mathematical terms used in mathematical sentences to mathematical objects.

[7] We would be careless, however, to simply assume that Benacerraf *endorses* the view that mathematical objects ground the truths of mathematical sentences. As is well-known, Benacerraf himself presents this way of understanding mathematical truth as a problem because it is incompatible with the causal theory of knowledge.

But there is also another way in which the Platonist can try to connect mathematical terms and their objects. He can do so via a correspondence theory of truth. For example, if one assumes that the sentences "There are at least three large cities older than New York" and "There are at least three perfect numbers greater than 17" are true because of how the world is, then it would appear that numbers (and other mathematical objects), just like cities, tables, chairs etc., are part of furniture of the world. So if the Platonist avails himself of the correspondence theory of truth, it seems that he can argue for a role of the objects of mathematics in grounding mathematical truths without the help of Quine's criterion.

Making sense of this view, however, requires a cashing out of the notion of correspondence. Specifically, what is needed is an account of truth that explicates the correspondence relation between mathematical sentences and the mathematical objects which (supposedly) provide the conditions for their truth. Notoriously, explicating the correspondence relation between mathematical objects and mathematical sentences is no easy feat. Correspondence accounts of truth have been unpopular *precisely* because the spelling out of the correspondence relation has been such an obstacle. And abstract objects, such as the objects of mathematics, pose the largest problem.

It is interesting that the truthmaker theory, the perhaps most sustained effort of developing a correspondence theory of truth, attempts to circumvent, rather than face that problem. Armstrong (2004), one of the strongest defenders of the truthmaker theory, argues that numbers are not needed as truthmakers. His reasons for this will prove instructive for our purposes.

Armstrong (2004: 5) characterizes his truthmaker theory like this: "[t]he idea of a truthmaker for a particular truth, then, is just some existent, some portion of reality, in virtue of which that truth is true." So the truthmaking relation is not a causal relation, rather, sentences (or for Armstrong, propositions), are true in virtue of what obtains in independent reality.

While it might be thought that the fact that the truthmaker relation is not a causal relation for Armstrong at least opens the door for objects without causal powers, he opts for an ontology without them.[8] His reason:

> To find truthmakers for certain truths, or sorts of truths, one wants to postulate entities that stand in various more or less complex relations of correspondence to these truths. But one wants these entities to be such that we can know, or at least have rational belief, that such entities exist. The entities must be such that they are epistemically accessible. (2004: 37)[9]

Why does Armstrong insist on positing only those entities that we can know about as truthmakers? As I see it, the reason for this is that if we don't, we are stripped of any legitimate reason for supposing that any sentences *are* true in the first place. It is thus unavoidable to tie talk of truth to epistemology in this way.[10]

While Armstrong (2004: 37) cashes out epistemic access mainly in terms of a causal, or nomic "link between the postulated truthmakers and the truths that we can fairly claim to know or to rationally believe," this link can be understood more widely, namely in terms of a reliabilist account of how we know about objects: For S to know that p, the knowledge must have been produced by a reliable process.[11]

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[8] See Armstrong, 2004: 112-118, for how this is done. I am not here endorsing the truthmaker theory.
[9] This is also the point that Benacerraf is worried about. On a causal theory of knowledge, it seems impossible that we could ever know the truths of mathematics.
[10] Note that this does *not* make the truth of a sentence dependent upon knowledge of it.
[11] Armstrong's text permits such an interpretation as well (see Armstrong, 2004: 37). Another point: cashing out Armstrong's point in terms of a reliabilist account of how we come to know objects also avoids the kind of reply the Platonist can make to the Eleaticicist: since mathematical objects are causally inert, they cannot make a contribution to the causal/nomic order of the world, but this does not mean that they might not be known in ways that don't involve such a contribution. However, on a reliabilist account of knowledge, these objects would not necessarily have to make a causal contribution in order for us to know about them.

Field (1989: 26) in couching Benacerraf's challenge (how we can know that mathematical sentences are true if the only way to know about objects is by causal interaction with them) in reliabilist language, has put the problem rather nicely:

> But Benacerraf's challenge … is to provide an account of the mechanisms that explain how our beliefs about these remote entities can so well reflect the facts about them. The idea is that *if it appears in principle impossible to explain this*, then that tends to *undermine* the belief in mathematical entities, *despite* whatever reason we might have for believing in them.

To me, Field's observation also gets at the heart of where the role of mathematical objects might ultimately be sought – and therefore, the way in which MND might be most plausibly understood: as the view that the objects of mathematics make no difference to how we come to *know* mathematical truths.

Before turning to this, we need to evaluate the success of the truth-version of MND vis-à-vis the observations made here. Do mathematical objects play a role in grounding the truth of mathematical claims? As we have seen, in order to win this argument, the Platonist needs to establish a connection between the mathematical terms used in mathematical sentences and their objects. He can do this either by invoking Quine's criterion (this will be discussed in section VI), or he has to provide an account of correspondence truth that explains the relationship between mathematical terms and mathematical objects. As we have seen, the most influential account of correspondence truth – the truthmaker theory, actually tries to circumvent having to do that, and for legitimate reasons: one first needs to explain how our beliefs about mathematical objects accurately reflects their properties. And for this, one in turn first needs to show that we can come to know about mathematical objects and their properties in a reliable way. Let us explore this path, and the last version of MND that it leads to.

V. MND and Mathematical Knowledge

We begin by providing a little more content to this epistemic version of MND. In the opening line to his 1994 (3), Azzouni describes mathematical practice in one succinct line: "The mathematician proves truths."[12] Later in the same work, Azzouni offers the following thought experiment: "Imagine that mathematical objects ceased to exist sometime in 1968. Mathematical work went on as usual. Why wouldn't it?"[13]

Why *would* mathematical practice go on as usual? According to Azzouni, "[g]iven standard mathematical practice, there seems to be *no* epistemic role for mathematical objects."[14] And on the next page: "It is not merely that mathematical objects do not seem causally involved in the processes we use to learn about their properties: it is that they seem to play no role at all."

In contrast, the objects of science clearly have an epistemic role. With some of these objects, we can interact in fairly direct causal ways (we can touch them, see them, etc.). Others – the more theoretical objects like electrons – we interact with by "designing instruments that are causally sensitive to their machinations" (1994: 53).[15] Mathematical objects do not play such a role. We don't interact with them, either directly, or indirectly via the use of instruments. Furthermore, I would add that we don't use mathematical objects to justify the truth or falsity of mathematical statments – that work is done by mathematical proof: it is the proof that is referred to in the justification of mathematical statements – not the mathematical objects (or their properties). Nor do mathematical objects or our interactions with them explain why we *believe* that some mathematical statements are

[12] Whether or not this is all a mathematician does will not concern us here.
[13] Azzouni, 1994: 56. Azzouni also discusses this thought experiment in his 2000. Baker (2003) addresses it as well.
[14] Azzouni, 1994: 55. Azzouni calls this the "epistemic role puzzle."
[15] Azzouni discusses this in more detail in his 1997.

true and others false. We believe in the truth of mathematical statements when (and because) a proof has been provided.[16]

Here, then, is a way we can formulate an epistemic version of MND: mathematical objects play *no role whatsoever* in the process by which we come to know mathematical truths. Therefore, we have no reason to believe that mathematical objects exist.

Does the Platonist have a reply to this? Well, at one point, mathematical Platonists claimed that we grasped mathematical objects via a faculty of mathematical intuition, and it was this grasping of the mathematical objects and their properties that provided justifications for believing in the truth of certain mathematical statements. This story may seem to best fit basic mathematical truths, such as that the number two is even or that circles are round. It might be argued that no proof for such truths appears to be necessary, yet, we seem to know them just the same. So, couldn't it be said that we know these truths because we intuitively grasp, or understand, the facts about these mathematical objects? If that were the case, this would delineate a role for mathematical objects after all: these objects would be involved in the epistemic process via the faculty of intuition.

A common - and quite reasonable - objection to the idea of resorting to a faculty of mathematical intuition as a way of explaining mathematical knowledge is that it doesn't actually explain anything, but instead, pushes the problem one step further back. After all, what is this mysterious faculty supposed to be? Thankfully, the current version of the position that we know mathematical truths via intuition has abandoned the view that intuition is a special faculty. Rather, intuition is understood as an ordinary cognitive *process*. For example, following in the footsteps of Katz and Bonjour, McEvoy (2004: 433) sees mathematical intuition as

> the fallible faculty of reason, representing the most basic elements of mathematical reality…

[16] I will say more about the role of proof in a moment.

> our ability to reason enables us to establish basic mathematical truths, and produces in us belief in these truths – without any causal or experiential connection between minds and abstract objects.

McEvoy argues that it is by accessing and examining mathematical *concepts*, not mathematical *objects*, that we come to know the truths about the abstract objects that these concepts represent. So, for instance, an analysis of the concepts of "four" and "composite" reveals "that it could not be the case that four was not composite" (2004: 433). When it comes to such basic mathematical truths, then, our capacity to reason does not have to be understood in terms of providing an elaborate proof, but rather, can be understood in terms of a simple conceptual analysis.

Mathematical concepts, for McEvoy, are "concepts of abstract objects." These concepts *represent* the mathematical objects and their properties.[17] So a direct causal link between us and abstract objects is not necessary for us to understand the truths about them. It might be objected that such a link is needed to acquire the mathematical concepts in the first place, but here McEvoy has a good response. In his view, "the Platonist can allow that our mathematical concepts originate from our *empirical* interactions with *physical* objects that approximate geometrical shapes, or that instantiate arithmetical structure." [2004: 433, emphasis mine][18]

Since physical shapes instantiate our concepts only approximately (e.g. there are no perfect circles), and since our arithmetical concepts outstrip the number of, and relations between, physical objects (e.g. not all numbers are instantiated), we "realize that our mathematical concepts…. must be concepts of abstract objects" (2004: 433). So as we become aware that circles are not perfectly round, we see that the mathematical concepts we have cannot really be concepts of objects in

[17] As the reader will no doubt have noticed, a lot is packed into this seemingly benign notion of "representation". We will unpack it shortly.

[18] A similar explanation is also offered by Resnik, 1997, chapter 9.

physical space. Therefore, we reason these concepts must represent the properties of abstract objects.

In this way, McEvoy believes he has offered an a priori and reliabilist account of how we gain mathematical knowledge of abstract objects – an account that does not require these objects to causally interact with us; and that does not presume any other experiential connection to the object (whatever that may come to). The account is a priori in the sense that mathematical knowledge is produced by a process of reasoning about concepts (and not about empirical objects). It is reliabilist because the reasoning process by which this knowledge is produced itself is reliable.[19] Lastly, as McEvoy says, his account of mathematical knowledge is also fallibilist. This is because mathematical knowledge can be revised based on a priori considerations.

The question now arises whether or not this new account of intuition succeeds in providing an epistemic role for mathematical objects. My view is that it does not. In order to see this, let us go through the argument McEvoy has presented step by step. The first step is that of concept acquisition. We learn about mathematical concepts by learning about empirical objects and their concepts. So far, so good. But no mathematical objects are involved in this process, by McEvoy's own description. The second step is that of reasoning about these concepts, or analyzing them, and coming to understand truths about them. This, too, does not require any mathematical objects.

Let me add one other ingredient to the process of gaining mathematical knowledge that I think is unobjectionable. In order to come to know more complex mathematical truths, we (or rather, the mathematicians) provide proofs. (I will here understand "proof" in a fairly loose sense, not requiring that each assumption and every logical step of the proof are written down. A proof, in this sense, is the sort of thing one finds in a standard mathematics textbook.)

[19] McEvoy, 2004: 428.

Still, so far, no part of the process described involves mathematical objects. In fact, that may be the *very reason* why this process of acquiring mathematical knowledge, as detailed by McEvoy, *is* reliable. It is reliable *because* it involves no access to, and no interaction with, abstract objects. And this is so *even if* we don't understand "access" or "interaction" in a causal way.

One might think that there's a way that mathematical objects enter the picture yet, albeit in a much more indirect way. McEvoy states, correctly, that mathematical concepts are not really instantiated in the physical world. The geometrical objects we encounter in physical space do not have exactly the properties that our mathematical formulas attribute to them, and the vast majority of numbers are not instantiated by collections of objects. Therefore, McEvoy reasons, these concepts must be *representations* of abstracta.

Let's play along with this for a moment. Let's assume, for the sake of the argument, that mathematical objects do exist, and that they indeed have the properties that we attribute to them. Let us also assume that we don't actually know this, because as McEvoy himself describes the process of gaining mathematical knowledge, these objects aren't involved: they are not interacted with *in any way*.

Here's how I want to press my objection: if all this is true, then what *reliable* way do we have of knowing that the concepts we have acquired from interaction with (mere) physical objects correctly *represent* the objects of mathematics and their properties? In other words, why doesn't Field's challenge reappear at this point? How do we explain that what we *believe* is true of mathematical objects (by abstraction from and extension of what we have learned by interaction with physical objects) really *is* true of mathematical objects?

We can certainly grant that physical circles, say, the circle that I carefully draw on the board with the help of a ruler, are (approximately) round. But how does it follow from this that the abstract object we take ourselves to refer to with the name of "circle" is round? How do we even know that among the abstract objects there *is* an object that represents the properties we have derived from studying (and idealizing) the properties of physical objects? This may sound like a very odd question to ask. But the only reason *why* it sounds odd is because that's what we mean by "circle". The word "circle" is what we use to name round objects. Our having settled on that name, however, doesn't force the existence of an abstract object that fits the description we have provided.

Let us pursue this objection by working with an example. The example will look intuitively dis-analogous, but further examination will prove this to be a chimera. This strategy will, I hope, reveal how strange the Platonist's picture really is. So, here goes. Imagine a philosophy student, call him Kitz. Kitz believes that there are abstract objects called "quirks", and that these quirks have certain properties, a, b, and c. Among these properties, Kitz believes, is the property of being spiffy. Kitz also believes, however, that we cannot interact with abstract objects in any way. We can interact only with physical objects and their properties. By abstracting away from the imperfections about these objects and their properties as we encounter them in the physical world, we realize, Kitz thinks, that there must be abstract objects which represent these properties perfectly.

Kitz now sets out to write a dissertation about quirks and their spiffyness. But his advisor, while a Platonist, has some doubts that Kitz will actually pass his defense. So he challenges Kitz to explain how he knows that quirks, that is, perfectly spiffy objects, are really among the realm of abstract objects. Kitz' spells his reasoning out like this: when he was a child, Kitz had received a very nice pair of red boots for Christmas, and he thought to himself, "now these are really spiffy." His new red boots were, of course, empirical objects, and he had causally interacted with them. Kitz also possessed the concept of spiffy. Spiffy, he had learned, was the property of being exceptionally cool, and this property most certainly applied to his new red boots.

Later on, having sat in metaphysics classes and learned about abstract objects, he came to believe that abstract objects are real, and that they instantiate properties he has acquired in the real world. So, he thought to himself, there must certainly be an abstract object that instantiates the property of spiffyness and, for lack of a better name, he named the object "quirk". (He would have liked to name the object "circle" but that name was already taken.)

Clearly, Kitz's reasoning is just an instance of bootstrapping. We can grant every single one of Kitz' claims: how he acquired the concept of spiffy, that he used the concept correctly, that his red boots were spiffy. In other words, we can grant that everything Kitz says about the physical world is true. We can also grant (just to be nice) that there are abstract objects. We can even allow Kitz to pick a hitherto unutilized sound, spelled out as "quirk", to name an object. Still, it just does not follow that there is an abstract object named by "quirk" which has the property of being spiffy.

Fast forward to Kitz's dissertation defense. (By sheer obstinacy, Kitz has gone on to complete his dissertation about quirks, and his advisor, tired of opposing Kitz' quest, has decided to just let him have it at the defense.) Here are the objections that are raised by his examination committee (all of whom, let's say, are mathematical Platonists).

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Examiner 1: "The case for mathematical objects is much stronger than your case for quirks. Take circles: we realize that physical circles aren't perfectly round, that is, that they only approximate the mathematical properties we attribute to circles. So, we reason that there must be an (abstract) object which does have the properties. This reasoning does not apply to quirks."

Kitz: "The way I see it, no object in the real world is exceptionally cool to the perfect degree. Only an abstract object could possess this quality perfectly. So the same reasoning *does* apply."

Examiner 2: "'Spiffy' is vaguely defined. Mathematical properties are not vague. So mathematical terms pick out abstract objects, but 'spiffy' does not succeed in picking out anything."

Kitz: "What is the argument for saying that vague terms cannot pick out abstract objects? Consider the pre-theoretical notion of "circle" as the Babylonians might have had it. They did not possess the formula we have for working out the area of a circle, so we might argue that they did not have a complete understanding of the exact properties of circles. Therefore, their concept of 'circle' was vaguely defined. But – I should think – when the Babylonians talked about circles they referred to the same abstract object we refer to when we talk about circles."

Examiner 3: "Mathematical objects have their properties necessarily. They cannot be otherwise. Quirks are *not* necessarily spiffy. They could be otherwise."

Kitz: "No, quirks *are* necessarily spiffy. I can't imagine them being any other way."

Examiner 3: "We are not talking about what we can or cannot imagine. We are talking about logical necessity."

Kitz: "I'm not talking about what we can imagine any more than you are."

Examiner 3: "But you are simply stipulating that quirks are necessarily spiffy."

Kitz: "Aren't you just stipulating as well? Consider basic mathematical truths – postulates, if you like. These are the sorts of things that are assumed in a proof, not the theorems they result in. Take one of Euclid's postulates, the one that says that the shortest distance between two points is a straight line. That's what we *mean* by the concept of a straight line. It's the shortest distance between two points. That's why it can't be otherwise. We have *stipulated* that that's what we mean by the words. Similarly, I stipulate that quirks are the sorts of things that are perfectly spiffy. So, it can't be otherwise." Digging in his heels, he adds: "prove me wrong!"

Examiner 3: "Stipulation does not bring into existence an object to which the term refers. By stipulation, you certainly don't bring into existence an object that is mind-independent, which we assume abstract objects to be.[20] Consider what Benacerraf has to say about stipulation: 'stipulation makes no connection between the propositions and their subject matter [the abstract object] –

[20] For a more detailed discussion of this claim, see Azzouni's 2000.

stipulation does not provide for truth.'[21] So how do you know that what you have stipulated to be true of the object *really is* true of the object?"

Kitz: "How do you know this with regard to the objects of mathematics? Really, I just don't see any difference between the case for mathematical objects and my object. So if you grant that mathematical objects exist, then you must grant the existence of quirks as well."

Well, I don't see a difference between the two cases either, and that was the point of the exercise, of course. However, I disagree with Kitz that he has made a case for quirks. Rather, I think matters are exactly the other way around. We cannot grant that a case has been made that anything Kitz says or believes about quirks is true of any abstract object. Such an object might exist, and it might not, but this makes no difference to Kitz's statements or beliefs. And the same is true for mathematical objects. The cases are exactly analogous, and so the epistemic version of MND stands up. Whether or not mathematical objects exist makes no difference to the process of how we come to know mathematical truths.

VI. The Role of Quine's Criterion

So, does this mean there is absolutely no role for mathematical objects? Before we can come to this conclusion, there is a loose end to consider. Above, we noted that according to McEvoy, mathematical concepts *represent* abstract objects. They *have* to represent abstract objects, he says, because they couldn't represent physical ones.

We can certainly grant that mathematical concepts don't represent *physical* objects, but why does it follow from this that they must represent *abstract* objects? The answer, again, lies in the application of Quine's criterion: the mathematical terms we use fall within the range of the

[21] Benacerraf 1973: 419.

(objectual) existential quantifier, and therefore, we seem to be committed to their existence. The real work of tying mathematical terms to mathematical objects, it appears, is done by Quine's criterion for ontological commitment. However indirect this role may be, unless the existence conferring role of Quine's criterion can be challenged, we will be saddled with the conclusion that mathematical objects do exist (even if their role is purely semantic).

But the proponent of MND is not out of options, for Quine's doctrine *has*, in fact, been challenged. For example, Yablo (2000: 304) has argued that the quantifiers must be understood metaphorically, not literally. Although this metaphorical use cannot always be paraphrased away, there's no reason to think that a metaphorical use of the quantifiers is ontologically committing.

More recently, Azzouni (2004: 54) has questioned the claim that the objectual existential quantifier must be understood as ontologically committing. The objectual existential quantifier in the object-language, he says, is read as ontologically committing by virtue of a domain of objects they range over (as we've seen). But that the objects in that domain *are objects we should be committed to* is determined by a reading of the existential quantifiers in the meta-language that provide the semantics for the object-language quantifiers. Nothing (in that meta-language or elsewhere) forces such a reading on us. If this is correct, then it is not true that the (objectual) existential quantifier must be understood as ontologically committing.

Whether or not we are committed to the entities that we quantify over is not going to be easily settled. What is needed is a careful evaluation of the arguments for and against Quine's criterion.[22] What is further needed is an evaluation of how we express our ontological commitments in English. Take, again, Benacerraf's mathematical sentence: "There are at least three perfect numbers greater than 17." When we utter that sentence in ordinary English, are we really expressing

[22] See, for instance, Azzouni, 2004, part I.

a commitment to the existence of three perfect numbers? Or are we simply committing ourselves to a truth (a truth we can justify by providing a mathematical proof)?

Rather than try to provide a quick, and therefore glib, answer to these questions in a few closing paragraphs, I want to end this paper with a challenge of my own: Would it really be such a bad thing if mathematical terms *didn't* refer? What, indeed is the reason for thinking that mathematical terms *must* represent *objects*?

This challenge isn't merely a verbal one. As I have tried to show in this paper, mathematical objects do not make a difference. And why should we think that our mathematical terms must represent objects when there is absolutely no role that these objects could play other than the sheer role of being the referents of our terms? If these objects did have a role, then yes, I think we *would* have to conclude that our mathematical terms refer (and that's indeed, the direction in which the argument should go). But we are not at all forced to this conclusion if mathematical objects do not make a difference.

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